DIFFERENTIAL EQUATION

SOLUTIONS TO 1ST ORDER DE

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Topic outcomes

At the end of the lesson, students must be able to:

1. Understand the two types of solutions for ordinary differential equations.
2. Solve basic differential equations using the method of separation of variables.
3. Recognize different forms of first-order differential equations and apply the appropriate solution method.

Solution of an ODE

- An **Ordinary Differential Equation** (ODE) is an equation that involves functions of a single variable and their derivatives. The variable is typically time or space, and the function represents a physical quantity, such as temperature, position, or velocity, that changes over time or space.

Importance of ODEs in Engineering and Science:

- ODEs are fundamental in modeling real-world systems across various fields of engineering and science. Whether it's predicting the motion of a pendulum, modeling the growth of a population, or understanding electrical circuits, ODEs play a crucial role in describing dynamic systems.

Solution of an ODE

Solution of an Ordinary Differential Equation

- Is a function y=f(x) that satisfies a given differential equation.

2 types of solutions

1. **General Solution**: The solution that contains arbitrary constants, representing the family of all possible solutions.
   - the entire set of solutions to a given differential equation

2. **Particular Solution**: A specific solution derived by applying initial or boundary conditions to the general solution.
   - member of a family of solutions to a differential equation that satisfies a particular initial condition

Solution of an ODE

Example of General Solution:

1. Find the general solution to the differential equation y'=2x.

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SOLUTION

Given the differential equation:

$\frac{dy}{dx} = 2x$

Step 1. Rewriting it as:

$dy = 2x\,dx$

Step 2. Integrate both sides:

$$\int dy = \int 2x\,dx$$

3.. The general solution is

$$y = x^2 + C$$

the general solution to the differential equation $\frac{dy}{dx} = 2x$ is:

$$y = x^2 + C$$

Solution of an ODE

Example of Particular Solution:

1. Find the particular solution to the differential equation y'=2x passing through the point (2,7)

Solution

Any function of the form $y = x^2 + C$ is a solution to this differential equation. To determine the value of C, we substitute the values $x = 2$ and $y = 7$ into this equation and solve for C:

$$y = x^2 + C$$
$$7 = 2^2 + C$$
$$= 4 + C$$
$$C = 3.$$

Therefore the particular solution passing through the point $(2,7)$ is $y = x^2 + 3$.

Solution of an ODE

Example of Particular Solution:

2. Find the particular solution to the differential equation $y' = 4x + 3$ passing through the point (1,7)

Solution of an ODE

solutions

step a. Find the general solution

$$\frac{dy}{dx} = 4x + 3$$

$$dy = (4x + 3)\,dx$$

$$\int dy = \int (4x + 3)\,dx$$

$$\boxed{y = 2x^2 + 3x + C}$$ general solution

Solution of an ODE

Step b. at $x = 1$ $y = 7$

solve for c using general solution. substitute x & y value

$$y = 2x^2 + 3x + c$$

$$7 = 2(1)^2 + 3(1) + c$$

$$7 = 2 + 3 + c$$

$$7 = 5 + c$$

$$7 - 5 = c$$

$$2 = c$$

$$c = 2$$

Substitute $c = 2$ to $y = 2x^2 + 3x + c$

$$\boxed{y = 2x^2 + 3x + 2}$$ → particular solution

Solution of an ODE

Example 3:
Consider the differential equation

$$\frac{dy}{dx} = 2x + e^x \text{ with the initial condition } y(0) = 1.$$

1.Find the general solution to the differential equation.
2. Determine the particular solution that satisfies the initial condition.

Solution of an ODE

Example 3:solution

Step 1. **Find the general solution:**

Integrate both sides of the differential equation:

$$\int \frac{dy}{dx}\,dx = \int (2x + e^x)\,dx$$

$$y = \int 2x\,dx + \int e^x\,dx$$

$$y = x^2 + e^x + C$$

Solution of an ODE

Example 3:solution

Step 2. **Determine the particular solution:**

Apply the initial condition $y(0) = 1$:

$$1 = 0^2 + e^0 + C$$

$$1 = 1 + C$$

Solving for C:

$$C = 0$$

Thus, the particular solution is:

$$y = x^2 + e^x$$

Solution of an ODE

Practice find the particular solution to the differential equation

Example 4

$$\frac{dy}{dx} = x^2 + 3 + e^x$$

given the initial condition y(0)=−1

Example 5

$$\frac{dy}{dx} = 4\sin(2x)$$

with the initial condition $y = 2$ when $x = \frac{\pi}{2}$.

Solution of an ODE

Example 4 solutions

1. **Integrate the differential equation:**

   Integrate both sides to find y:

$$\int \frac{dy}{dx}\, dx = \int (x^2 + 3 + e^x)\, dx$$

   Compute the integrals:

$$y = \int x^2\, dx + \int 3\, dx + \int e^x\, dx$$

$$y = \frac{x^3}{3} + 3x + e^x + C$$

2. **Determine the particular solution:**

   Use the initial condition $y(0) = -1$ to find C:

$$-1 = \frac{0^3}{3} + 3(0) + e^0 + C$$

$$-1 = 0 + 0 + 1 + C$$

$$-1 = 1 + C$$

$$C = -1 - 1$$

$$C = -2$$

$$y = \frac{x^3}{3} + 3x + e^x - 2$$

Solution of an ODE

Example 5 solutions Step 1

$$\frac{dy}{dx} = 4\sin(2x)$$

$$\int \frac{dy}{dx}\, dx = \int 4\sin(2x)\, dx$$

To integrate $4\sin(2x)$, use a substitution method. Let $u = 2x$, so $du = 2\, dx$,

$$dx = \frac{du}{2}$$

$$\int 4\sin(2x)\, dx = 4\int \sin(u) \cdot \frac{du}{2}$$

$$= 2\int \sin(u)\, du$$

= - 2 cos (u) + C

$$y = -2\cos(2x) + C$$

Solution of an ODE

Example 5 solutions

Step 2 **Determine the particular solution:**

Use the initial condition $y = 2$ when $x = \frac{\pi}{2}$:

$$2 = -2\cos\left(2 \cdot \frac{\pi}{2}\right) + C$$

$$2 = -2\cos(\pi) + C$$

Since $\cos(\pi) = -1$:

$$2 = -2(-1) + C$$

$$2 = 2 + C$$

$$C = 2 - 2$$

$$C = 0$$

Thus, the particular solution is:

$$y = -2\cos(2x)$$