CALCULUS 2

ANTIDERIVATIVES (INTEGRAL)

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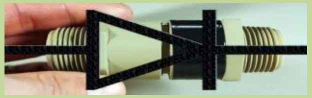
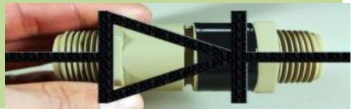


OBJECTIVES



- integrate functions of the of the different inverse trigonometric functions
- identify the different hyperbolic functions;
- find the integral of given hyperbolic functions;
- determine the difference between the integrals of hyperbolic functions; and
- evaluate integrals involving hyperbolic functions.

INTEGRAL CALCULUS

Review

- Recall derivatives of inverse trig functions

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad |u|<1$$

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\sec^{-1}u = \frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}, \quad |u|<1$$

Integrals Using Same Relationships

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left|\frac{u}{a}\right| + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \text{for } a > 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \qquad \text{for } a > 0$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C \qquad \text{for } a > 0$$

INTEGRAL CALCULUS

. If a=1, you have:

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{du}{1+u^2} = \tan^{-1} u + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}|u| + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}|x| + C$$

Identifying Patterns

- For each of the integrals below, which inverse trig function is involved?

$$\int \frac{4dx}{13+16x^2}$$

$$\int \frac{dx}{x\sqrt{25x^2-4}}$$

$$\int \frac{dx}{\sqrt{9-x^2}}$$

Warning

- Many integrals look like the inverse trig forms
- Which of the following are of the inverse trig forms?

$$\int \frac{x\,dx}{1+x^2} \qquad \int \frac{x\,dx}{\sqrt{1-x^2}}$$

$$\int \frac{dx}{1+x^2} \qquad \int \frac{dx}{\sqrt{1-x^2}}$$

If they are not, how are they integrated?

Try These

- Look for the pattern or how the expression can be manipulated into one of the patterns

$$\int \frac{8dx}{1+16x^2}$$

$$\int \frac{dx}{\sqrt{-4x^2+4x+15}}$$

$$\int \frac{x\,dx}{\sqrt{1-25x^2}}$$

$$\int \frac{x-5}{\sqrt{x^2-10x+16}}\,dx$$

Completing the Square

- Often a good strategy when quadratic functions are involved in the integration

$$\int \frac{dx}{x^2 + 2x + 10}$$

- Remember … we seek $(x - b)^2 + c$
  - Which might give us an integral resulting in the arctan function

Completing the Square

- Try these

$$\int \frac{dx}{x^2 + 4x + 13}$$

$$\int \frac{2}{\sqrt{-x^2 + 4x}}\, dx$$

Completing the Square

- Try these

$$\int \frac{dx}{x^2 + 4x + 13}$$

$x^2 + 4x + 13 \rightarrow x^2 + 4x + 4 + 9$

$\downarrow$

$(x + 2)^2 + 9$

$$\int \frac{2}{\sqrt{-x^2 + 4x}}\, dx$$

$\rightarrow -(x^2 - 4x + 4) + 4 \rightarrow -(x-2)^2 + 4$

Examples

Evaluate $\int \frac{dx}{1+3x^2}$.

Solution. Substituting

$$u = \sqrt{3}x, \quad du = \sqrt{3}\,dx$$

$$\int \frac{dx}{1+3x^2} = \frac{1}{\sqrt{3}} \int \frac{du}{1+u^2}$$

$$\int \frac{du}{1+u^2} = \tan^{-1} u + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} u + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}(\sqrt{3}x) + C$$

Examples

Evaluate $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$.

Solution. Substituting

$$u = e^x, \quad du = e^x\,dx$$

yields

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int \frac{du}{\sqrt{1-u^2}}$$

$$= \sin^{-1} u + C$$

$$= \sin^{-1}(e^x) + C$$

Examples

Evaluate $\int \frac{dx}{\sqrt{2-x^2}}$.

$u = x$ and $a = \sqrt{2}$

$$\int \frac{dx}{\sqrt{2-x^2}} = \sin^{-1}\frac{x}{\sqrt{2}} + C$$

4. $\int \frac{x^2\,dx}{1+x^6}; \; u = x^3$

$$\frac{1}{3}\int \frac{du}{1+u^2}$$

$$= \frac{1}{3}\tan^{-1}(x^3) + C.$$

5. $$\int \frac{dx}{x\sqrt{1-(\ln x)^2}};$$

$$u = \ln x,$$

$$\int \frac{1}{\sqrt{1-u^2}}du:$$

$$= \sin^{-1}(\ln x) + C.$$

6. $$\int \frac{dx}{9+x^2} = \frac{1}{3}\arctan\frac{x}{3} + C$$

a = 3
u = x

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

INTEGRAL CALCULUS

61. (a) $\displaystyle\int \frac{dx}{\sqrt{9-x^2}}$ (b) $\displaystyle\int \frac{dx}{5+x^2}$ (c) $\displaystyle\int \frac{dx}{x\sqrt{x^2-\pi}}$

61. (a) $\sin^{-1}(x/3)+C$. (b) $(1/\sqrt{5})\tan^{-1}(x/\sqrt{5})+C$. (c) $(1/\sqrt{\pi})\sec^{-1}|x/\sqrt{\pi}|+C$.