CALCULUS 2

ANTIDERIVATIVES (INTEGRAL)

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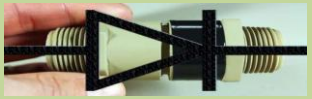

OBJECTIVES

- identify the different hyperbolic functions;
- find the integral of given hyperbolic functions;
- determine the difference between the integrals of hyperbolic functions; and
- evaluate integrals involving hyperbolic functions.

INTEGRAL CALCULUS

Integration of Hyperbolic Functions

INTEGRAL CALCULUS

Definitions:

1. $\sinh x = \dfrac{e^x - e^{-x}}{2}$

2. $\cosh x = \dfrac{e^x + e^{-x}}{2}$

3. $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

4. $\coth x = \dfrac{1}{\tanh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

5. $\sec hx = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

6. $\csc hx = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$

INTEGRAL CALCULUS

Differentiation Formulas

$1.d(\sinh u) = \cosh u du$

$2.d(\cosh u) = \sinh u du$

$3.d(\tanh u) = \operatorname{sec} h^2 u du$

$4.d(\coth u) = -\operatorname{csc} h^2 u du$

$5.d(\operatorname{sec} hu) = -\operatorname{sec} hu \tanh u du$

$6.\mathrm{d}(\operatorname{csc} \mathrm{hu}) = -\operatorname{csc} \mathrm{hu} \coth \mathrm{udu}$

INTEGRAL CALCULUS

1. $\int \sinh u du = \cosh u + C$

2. $\int \cosh u du = \sinh u + C$

3. $\int \sec h^2 u du = \tanh u + C$

4. $\int \csc h^2 du = -\coth u + C$

5. $\int \sec hu \tanh u du = -\sec hu + C$

6. $\int \csc hu \coth u du = -\csc hu + C$

7. $\int \tanh u du = \int \frac{\sinh u du}{\cosh u}$
$= \ln(\cosh u) + c$

8. $\int \coth u du = \int \frac{\cosh u du}{\sinh u}$
$= \ln(\sinh u) + c$

9. $\int \sec hu du = 2\tan^{-1}(e^u) + C$
$= \tan^{-1}(\sinh u) + C$

10. $\int \csc hu du = \ln\left|\tanh\frac{u}{2}\right| + C$

1) $\int \sinh(1-3x)\,dx$

$u = 1-3x$

$du = -3\,dx$

$\frac{du}{-3} = dx$

$= \int \sinh u \frac{du}{-3}$

$= -\frac{1}{3}\int \sinh u\,du$

$= -\frac{1}{3}\cosh u + c$

$= -\frac{1}{3}\cosh(1-3x) + c$

2. $\int e^{2x}\cosh e^{2x}\,dx$

$u = e^{2x}$

$du = 2e^{2x}\,dx$

$\frac{du}{2} = e^{2x}\,dx$

$= \int \cosh e^{2x}\, e^{2x}\,dx$

$= \int \cosh u \frac{du}{2}$

$= \frac{1}{2}\int \cosh u\,du$

$= \frac{1}{2}\sinh u + c$

$= \frac{1}{2}\sinh e^{2x} + c$

3) $\int \operatorname{sech}^2(2x+6)\,dx$

$u = 2x+6$

$du = 2\,dx$

$\frac{du}{2} = dx$

$= \int \operatorname{sech}^2 u \frac{du}{2}$

$= \frac{1}{2}\int \operatorname{sech}^2 u\,du$

$= \frac{1}{2}\tanh u + c$

$= \frac{1}{2}\tanh(2x+6) + c$

4) $\int \operatorname{csch}^2(x+3)\,dx$

$u = x+3$

$du = dx$

$= \int \operatorname{csch}^2 u\,du$

$= -\coth u + C$

$\boxed{= -\coth(x+3) + C}$

5) $\int \operatorname{sech}(3x)\tanh(3x)\,dx$

$u = 3x$

$du = 3\,dx$

$\frac{du}{3} = dx$

$= \int \operatorname{sech} u \tanh u \frac{du}{3}$

$= \frac{1}{3}\int \operatorname{sech} u \tanh u\,du$

$= \frac{1}{3} -\operatorname{sech} u + C$

$\boxed{= -\frac{1}{3}\operatorname{sech} 3x + C}$

$$\int \tanh\left(\frac{1}{3}x\right)dx$$

$$u = \frac{1}{3}x$$

$$du = \frac{1}{3}dx$$

$$3\,du = dx$$

$$= \int \tanh u \; 3du$$

$$= 3\int \tanh u \, du$$

$$= 3\ln(\cosh u) + C$$

$$\boxed{= 3\ln\left(\cosh\frac{1}{3}x\right) + C}$$

INTEGRAL CALCULUS

$$\int \frac{\cosh x}{2+3\sinh x}dx.$$

$$\int \frac{\cosh x}{2+3\sinh x}dx = \int \frac{\frac{du}{3}}{u} :$$

$u = 2+3\sinh x,$

$du = 3\cosh x dx$

$\cosh x dx = \frac{du}{3}$

$$= \frac{1}{3}\ln|u| + C :$$

$$= \frac{1}{3}\ln|2+3\sinh x| + C.$$