CALCULUS 2

INTRODUCTION TO ANTIDERIVATIVE

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Differential calculus finds numerous applications in real life scenarios. Here are a few examples:

1.**Physics**:

1. **Motion Analysis**: Calculus is used to analyze the motion of objects. For instance, it helps determine velocity and acceleration of moving objects, which is crucial in designing transportation systems, predicting trajectories of projectiles, and understanding the behavior of celestial bodies.
2. **Electric Circuits**: Differential equations are used to model electric circuits, enabling engineers to design and optimize electronic devices such as computers, smartphones, and communication systems.

2.**Engineering**:

1. **Mechanical Systems**: Differential calculus is used to analyze forces, stresses, and deformations in mechanical systems like bridges, buildings, and machinery. It helps engineers design structures that can withstand various loads and environmental conditions.
2. **Control Systems**: Calculus is applied in designing control systems for automobiles, aircraft, and industrial processes. It helps ensure stability, accuracy, and efficiency in controlling the behavior of dynamic systems.

3.**Economics and Finance**:

1. **Optimization**: Differential calculus is used to optimize functions representing costs, revenues, and profits in economics and finance. It aids in determining the best strategies for maximizing profits, minimizing costs, and allocating resources efficiently.
2. **Interest Rates**: Differential equations are employed to model interest rates and their effects on investments, loans, and financial markets. They help in analyzing and predicting changes in interest rates and their impact on economic activities.

1. **Biology and Medicine**:
    1. **Pharmacokinetics**: Differential equations are used to model the absorption, distribution, metabolism, and excretion of drugs in the body. This helps in designing effective drug dosages and understanding drug interactions and side effects.
    2. **Neuroscience**: Calculus is utilized to study the dynamics of neural networks in the brain, aiding in understanding cognitive processes, learning mechanisms, and neurological disorders.
2. **Environmental Science**:
    1. **Pollution Dispersion**: Differential equations are applied to model the dispersion of pollutants in the atmosphere, oceans, and water bodies. They help in assessing environmental impacts, designing pollution control measures, and formulating environmental policies.
    2. **Climate Modeling**: Calculus is used in climate modeling to study the dynamics of Earth's atmosphere, oceans, and ecosystems. It helps in predicting climate change, extreme weather events, and their consequences on global ecosystems and human societies.

The integral calculus has a wide range of real-life applications across various fields. Here are some examples:

1.Physics and Engineering:

1. **Area and Volume Calculations**: Integrals are used to calculate areas under curves and volumes of complex shapes. For example, in civil engineering, integrals help determine the volume of materials needed to construct structures like bridges or dams.
2. **Mechanics**: Integrals are used to calculate work done by a force, which is essential in understanding energy transfer and mechanical systems' behavior. For instance, integrating force over distance gives the work done in moving an object.

2.Economics and Finance:

1. **Consumer and Producer Surplus**: Integrals are used to calculate consumer and producer surplus in economics, which helps in understanding market efficiency and welfare economics.
2. **Revenue and Cost Analysis**: Integrals help in analyzing revenue and cost functions to determine optimal pricing strategies, profit maximization, and cost minimization.

3.Probability and Statistics:

1. **Probability Density Functions**: Integrals are used to calculate probabilities in continuous probability distributions. For example, in statistics, the integral of a probability density function over a certain interval gives the probability of a random variable falling within that interval.
2. **Expected Value Calculations**: Integrals are used to calculate expected values of random variables, which are crucial in decision-making and risk analysis.

1. **Biology and Medicine**:
    1. **Medical Imaging**: Integrals are used in medical imaging techniques like MRI (Magnetic Resonance Imaging) and CT (Computed Tomography) scans to reconstruct images from measured signals. Integrals are involved in processes like Radon transforms and filtered back-projection.
    2. **Drug Concentration Analysis**: Integrals are used to model drug concentration in the bloodstream over time, aiding in drug dosage determination and understanding pharmacokinetics.
2. **Environmental Science**:
    1. **Environmental Impact Assessment**: Integrals are used to calculate various environmental parameters such as pollutant concentrations, biodiversity indices, and habitat fragmentation, aiding in environmental impact assessments and conservation planning.
    2. **Ecosystem Services Valuation**: Integrals are used to estimate the value of ecosystem services such as carbon sequestration, water purification, and crop pollination, assisting in natural resource management and policy-making.
3. **Finance**:
    1. **Option Pricing**: Integrals are used in finance to price options and other financial derivatives. Techniques like the Black-Scholes formula involve the use of integrals to calculate option prices.
    2. **Portfolio Optimization**: Integrals are used to calculate portfolio risk and return, aiding in portfolio optimization and asset allocation strategies.

OBJECTIVES

At the end of the lesson the students are expected to:

- ◆ know the relationship between differentiation and integration;
- ◆ identify and explain the different parts of the integral operation; and
- ◆ perform basic integration by applying the power formula and the properties of the indefinite integrals.

INTEGRAL CALCULUS

DEFINITION: ANTIDERIVATIVE (INTEGRAL)

A function *F* is called an *antiderivative (or integral)* of the function *f* on a given open interval if *F'(x) = f(x)* for every value of *x* in the interval.

For example, the function $F(x) = \frac{1}{3}x^2$ is an antiderivative of f(x) = x^2 on interval $(-\infty, +\infty)$ because for each *x* in this interval

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$$F'(x) = \frac{d}{dx}\left[\frac{1}{3}x^3\right] = x^2 = f(x)$$

However, $F(x) = \frac{1}{3}x^3$ is not the only antiderivative of *f* on this interval. If we add any constant C to $\frac{1}{3}x^3$, then the function

$$G'(x) = \frac{d}{dx}\left[\frac{1}{3}x^3 + C\right] = x^2 + 0 = f(x)$$

INTEGRAL CALCULUS

In general, once any single antiderivative is known, the other antiderivatives can be obtained by adding constants to the known derivative. Thus,

$$\frac{1}{3}x^3, \quad \frac{1}{3}x^3+2, \quad \frac{1}{3}x^3-5, \quad \frac{1}{3}x^3+\sqrt{2}$$

are all antiderivatives of $f(x) = x^2$.

Theorem: If *F(x) is* any antiderivative of *f(x)* on an open interval, then for any constant C the function *F(x)+C* is also an antiderivative on that interval. Moreover, each antiderivative of *f(x)* on the interval can be expressed in the form *F(x)+C* by choosing the constant *C* appropriately.

INTEGRAL CALCULUS

DEFINITION: THE INDEFINITE INTEGRAL

The process of finding antiderivatives is called ***antidifferentiation or integration.*** Thus, if

$$\frac{d}{dx}[F(x)] = f(x)$$

then integrating (or antidifferentiating) the function *f(x)* produces an antiderivative of the form *F(x)+C.* To emphasize this process, we use the following *integral notation*

$$\int f(x)dx = F(x) + C$$

INTEGRAL CALCULUS

DEFINITION: THE INDEFINITE INTEGRAL

The process of finding antiderivatives is called *antidifferentiation* or *integration*. Thus, if

$$\frac{d}{dx}[F(x)] = f(x) \tag{1}$$

then *integrating* (or *antidifferentiating*) the function $f(x)$ produces an antiderivative of the form $F(x) + C$. To emphasize this process, Equation (1) is recast using *integral notation*,

$$\int f(x)\,dx = F(x) + C \tag{2}$$

where C is understood to represent an arbitrary constant. It is important to note that (1) and (2) are just different notations to express the same fact. For example,

$$\int x^2\,dx = \tfrac{1}{3}x^3 + C \quad \text{is equivalent to} \quad \frac{d}{dx}\left[\tfrac{1}{3}x^3\right] = x^2$$

Note that if we differentiate an antiderivative of $f(x)$, we obtain $f(x)$ back again. Thus,

$$\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x) \tag{3}$$

INTEGRAL CALCULUS

where:

- The expression is $\int f(x)dx$ called an *indefinite integral.*
- $\int$ is called an integral sign
- the function $f(x)$ is called the integrand
- and the constant C is called the constant of integration
- dx indicates that x is the variable of integration.

$\int f(x)\,dx = F(x) + C$ read as "The integral of f(x) with respect to x is equal to F(x) plus a constant."

INTEGRAL CALCULUS

The differential symbol, *dx*, in the differentiation and antidifferentiation operations

$$\frac{d}{dx}[\ ] \quad \text{and} \quad \int [\ ]\, dx$$

serves to identify the independent variable. If an independent variable other than *x* is used,
say *t* , then the notation must be adjusted appropriately. Thus

$$\frac{d}{dt}[F(t)] = f(t) \quad \text{and} \quad \int f(t)\, dt = F(t) + C$$

are equivalent statements.

INTEGRAL CALCULUS

Here are some examples of derivative formulas and their equivalent integration formulas:

DERIVATIVE FORMULA	EQUIVALENT INTEGRATION FORMULA
$\frac{d}{dx}[x^3] = 3x^2$	$\int 3x^2\,dx = x^3 + C$
$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$	$\int \frac{1}{2\sqrt{x}}\,dx = \sqrt{x} + C$
$\frac{d}{dt}[\tan t] = \sec^2 t$	$\int \sec^2 t\,dt = \tan t + C$
$\frac{d}{du}[u^{3/2}] = \frac{3}{2}u^{1/2}$	$\int \frac{3}{2}u^{1/2}\,du = u^{3/2} + C$

INTEGRAL CALCULUS

For simplicity, the dx is sometimes absorbed into the integrand. For example,

$$\int 1\,dx \quad \text{can be written as} \quad \int dx$$

$$\int \frac{1}{x^2}\,dx \quad \text{can be written as} \quad \int \frac{dx}{x^2}$$

INTEGRAL CALCULUS

GENERAL STEPS IN SOLVING INTEGRAL

Original integral	$\int 3x\,dx = 3\int x\,dx$	Constant Multiple Rule
Rewrite	$= 3\int x^1\,dx$	Rewrite x as x^1.
Integrate	$= 3\left(\frac{x^2}{2}\right) + C$	Power Rule $(n = 1)$
Simplify	$= \frac{3}{2}x^2 + C$	Simplify.

PROPERTIES OF THE INDEFINITE INTEGRAL

Our first properties of antiderivatives follow directly from the simple constant factor, sum, and difference rules for derivatives.

5.2.3 THEOREM *Suppose that $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ and $g(x)$, respectively, and that c is a constant. Then:*

(a) *A constant factor can be moved through an integral sign; that is,*

$$\int cf(x)\,dx = cF(x) + C$$

(b) *An antiderivative of a sum is the sum of the antiderivatives; that is,*

$$\int [f(x) + g(x)]\,dx = F(x) + G(x) + C$$

(c) *An antiderivative of a difference is the difference of the antiderivatives; that is,*

$$\int [f(x) - g(x)]\,dx = F(x) - G(x) + C$$

INTEGRAL CALCULUS

The statements in Theorem 5.2.3 can be summarized by the following formulas:

$$\int cf(x)\,dx = c\int f(x)\,dx \tag{4}$$

$$\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx \tag{5}$$

$$\int [f(x)-g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx \tag{6}$$

INTEGRATION FORMULA

$$\int dx = x + C$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$$

To integrate a power of x (other than −1), add 1 to the exponent and divide by the new exponent.

INTEGRAL CALCULUS

Here are some examples:

$$\int x^2\,dx = \frac{x^3}{3} + C \qquad \boxed{r = 2}$$

$$\int x^3\,dx = \frac{x^4}{4} + C \qquad \boxed{r = 3}$$

$$\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx = \frac{x^{-5+1}}{-5+1} + C = -\frac{1}{4x^4} + C \qquad \boxed{r = -5}$$

$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \tfrac{2}{3}x^{\frac{3}{2}} + C = \tfrac{2}{3}(\sqrt{x}\,)^3 + C \qquad \boxed{r = \tfrac{1}{2}}$$

- **Example** 2. EVALUATE

$$\int (3x^6 - 2x^2 + 7x + 1)\, dx =$$

SOLUTION

$$= 3\int x^6\, dx - 2\int x^2\, dx + 7\int x\, dx + \int 1\, dx$$

$$= \frac{3x^7}{7} - \frac{2x^3}{3} + \frac{7x^2}{2} + x + C$$

INTEGRAL CALCULUS

EXAMPLE

Evaluate the following integral.

1. $\int x^4 \, dx$
2. $\int (3x^2 - 6x + 7)dx$
3. $\int (2a^2x^2 - b^3)^2 dx$
4. $\int \sqrt{y}\left(\frac{1-\sqrt[3]{y}}{2y^{\frac{2}{3}}}\right) dy$

EXAMPLE

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4. $\int \sqrt{y}\left(\frac{1-\sqrt[3]{y}}{2y^{\frac{2}{3}}}\right)dy$

$= \frac{x^5}{5} + C$

$= \frac{3x^3}{3} - \frac{6x^2}{2} + 7x + C = x^3 - 3x^2 + 7x + C$

3) $\int [(2a^2x^2 - b^3)(2a^2x^2 - b^3)]dx$

$(4a^4x^4 - 2a^2b^3x^2 - 2a^2b^3x^2 + b^6)dx$

$\int (4a^4x^4 - 4a^2b^3x^2 + b^6)dx$

$= \int 4a^4x^4dx - \int 4a^2b^3x^2dx + \int b^6dx$

$= 4a^4\int x^4dx - 4a^2b^3\int x^2dx + b^6\int dx$

$= 4a^4\left[\frac{x^5}{5}\right] - 4a^2b^3\left[\frac{x^3}{3}\right] + b^6(x) + C$

$= \frac{4a^4x^5}{5} - \frac{4a^2b^3x^3}{3} + b^6x + C$

$$4. \int \sqrt{y}\left(\frac{1-\sqrt[3]{y}}{2y^{\frac{2}{3}}}\right) dy$$

$$\int y^{\frac{1}{2}}\left(\frac{1-y^{1/3}}{2y^{2/3}}\right) dy$$

$$= \int \left(\frac{1-y^{1/3}}{y^{-1/2} \cdot 2y^{2/3}}\right) dy$$

$$= \left(\frac{1-y^{1/3}}{2(y^{-\frac{1}{2}+\frac{2}{3}})}\right) dy$$

$$= \left(\frac{1-y^{1/3}}{2y^{1/6}}\right) dy$$

$$= \left(\frac{1}{2y^{1/6}} - \frac{y^{1/3}}{2y^{1/6}}\right) dy$$

$$y^{\frac{1}{3}} \cdot y^{-\frac{1}{6}}$$

$$= \int \left(\frac{1}{2}y^{-1/6} - \frac{1}{2}y^{1/6}\right) dy$$

$$= \frac{1}{2}\int y^{-1/6} dy - \frac{1}{2}\int y^{1/6} dy$$

$$= \frac{1}{2}\left[\frac{y^{-\frac{1}{6}+1}}{-\frac{1}{6}+1}\right] - \frac{1}{2}\left[\frac{y^{1/6+1}}{\frac{1}{6}+1}\right] + C$$

$$= \frac{1}{2}\left[\frac{y^{\frac{5}{6}}}{\frac{5}{6}}\right] - \frac{1}{2}\left[\frac{y^{7/6}}{\frac{7}{6}}\right] + C$$

$$= \frac{6}{10}y^{5/6} - \frac{6}{14}y^{7/6} + C$$

$$= \boxed{\frac{3}{5}y^{5/6} - \frac{3}{7}y^{7/6} + C}$$

Practice sets

Evaluate the following integral.

1. $\int \left(5x^3 - 2x^2 + 3\right) dx$

2. $\int \left(y^{\frac{2}{3}} - 4y^{-\frac{1}{5}} + 4 \right) dy$

3. $\int \frac{dt}{\sqrt[3]{t^2}}$

4. $\int \left(\frac{7}{z^{\frac{3}{4}}} - 4\sqrt{z} + \sqrt[3]{z} \right) dz$

5. $\int (t + 5a)^3 \, dt$

6. $\int \frac{z^3 + 1}{z + 1} dz$

7. $\int \left(x^{\pi} + 2x^{e}\right) dx$

8. $\int \sqrt[3]{x} \left(\frac{7x - 4\sqrt{x}}{\sqrt[5]{x}} \right) dx$

9. $\int \sqrt[3]{8x^3 + 36x^2 + 54x + 27}\, dx$

10. $\int \sqrt{m^6 + 2 + m^{-6}}\, dm$