CALCULUS 2

ANTIDERIVATIVES (INTEGRAL)

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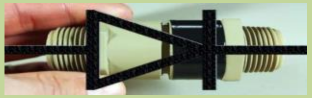
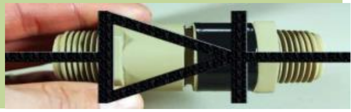


OBJECTIVES



At the end of the lesson the students are expected to:

- identify an integrand that can be integrated using simple substitution;
- perform integration using the generalized power formula;
- relate integration by power formula to the generalized integration formula;
- and consider and use the “introduction of neutralizing/correction factor” as an alternative technique of integration.

INTEGRATION BY SUBSTITUTION

- A technique called ***substitution***, that can often be used to transform complicated integration problems into simpler ones.

u-SUBSTITUTION

The method of substitution can be motivated by examining the chain rule from the viewpoint of antidifferentiation. For this purpose, suppose that *F* is an antiderivative of *f* and that *g* is a differentiable function. The chain rule implies that the derivative of *F(g(x))* can be expressed a $\frac{d}{dx}[F(g(x))] = F'(g(x))g'(x)$ which we can write in integral form as $\int F'(g(x))g'(x)dx = F(g(x)) + C$

or since F is an antiderivative of f,

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C \qquad (2)$$

For our purposes it will be useful to let $u = g(x)$ and to write $du/dx = g'(x)$ in the differential form $du = g'(x)\,dx$. With this notation (2) can be expressed as

$$\int f(u)\,du = F(u) + C \qquad (3)$$

The process of evaluating an integral of form (2) by converting it into form (3) with the substitution

$$u = g(x) \quad \text{and} \quad du = g'(x)\,dx$$

is called the ***method of u-substitution***.

The generalized power formula therefore is:

$$\int [f(u)]^n \, d[f(u)] = \frac{[f(u)]^{n+1}}{n+1} + C; \qquad n \neq -1$$

Guidelines for u-Substitution

Step 1. Look for some composition $f(g(x))$ within the integrand for which the substitution

$$u = g(x), \quad du = g'(x)\,dx$$

produces an integral that is expressed entirely in terms of u and its differential du. This may or may not be possible.

Step 2. If you are successful in Step 1, then try to evaluate the resulting integral in terms of u. Again, this may or may not be possible.

Step 3. If you are successful in Step 2, then replace u by $g(x)$ to express your final answer in terms of x.

▸ Example 1 Evaluate $\int (x^2 + 1)^{50} \cdot 2x \, dx$.

Solution. If we let $u = x^2 + 1$, then $du/dx = 2x$, which implies that $du = 2x \, dx$. Thus, the given integral can be written as

$$\int (x^2 + 1)^{50} \cdot 2x \, dx = \int u^{50} \, du = \frac{u^{51}}{51} + C = \frac{(x^2 + 1)^{51}}{51} + C \blacktriangleleft$$

INTEGRAL CALCULUS

2. Evaluate $\int \frac{dx}{\left(\frac{1}{3}x-8\right)^5}$

Solution:

$$u = \tfrac{1}{3}x - 8$$
$$du = \tfrac{1}{3}\,dx \text{ or } dx = 3\,du$$

$$= \int \frac{3\,du}{u^5}$$

$$= 3\int u^{-5}\,du$$

$$= -\frac{3}{4}u^{-4} + C \quad = -\frac{3}{4}\left(\frac{1}{3}x - 8\right)^{-4} + C$$

3. **Evaluate** $\int x\sqrt{1-x}\,dx$.

INTEGRAL CALCULUS

3. **Evaluate** $\int x\sqrt{1-x}\,dx.$

Solution:

Let u = 1 – x ; also x = 1 - u

du = - dx

Or dx = - du

Substitution yields

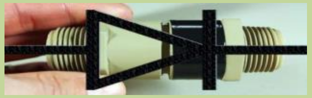
$$= \int (1-u)(\sqrt{u})\text{-du}$$

$$= -\int (1-u)(u^{\frac{1}{2}})\ \text{du}$$

$$= -\int (u^{\frac{1}{2}} - u^{\frac{3}{2}})\ \text{du}$$

$$= -\frac{2}{3}u^{\frac{3}{2}} + \frac{2}{5}u^{\frac{5}{2}} + \text{C}$$

$$= -\frac{2}{3}(1-x)^{\frac{3}{2}} + \frac{2}{5}(1-x)^{\frac{5}{2}} + \text{C}$$



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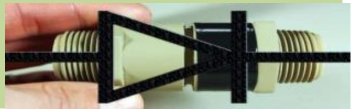

At the end of the lesson the students are expected to:

- Define the natural logarithmic function;
- Illustrate solutions on problem yielding to natural logarithmic function;
- Derive particular formulas involving integrals relative to natural logarithmic function; and
- evaluate problems involving natural logarithmic functions.

INTEGRAL CALCULUS

$$Recall:\ d\left(\ln u\right) = \frac{du}{u}$$

Evaluate the integral of each side

$$\int \frac{du}{u} = \int d\left(\ln u\right)$$

OR

$$\int \frac{du}{u} = \ln\left|u\right| + C$$

EXAMPLE:

1. $\int \frac{3}{2x-7}\ dx$

$u = 2x - 7$
$du = 2\ dx$
$dx = du/2$

$= \int \frac{3}{u}\ \left(\frac{du}{2}\right)$

$= \frac{3}{2} \int \frac{du}{u}$

$= \frac{3}{2} \ln |u| + C$

$= \frac{3}{2} \ln |2x - 7| + C$

INTEGRAL CALCULUS

$u = x^2 + x + 5 \quad du = (2x + 1)\, dx$

$dx = \frac{du}{2x+1}$

2. $\int \frac{4x + 2}{x^2 + x + 5} dx$

$= \int \frac{4x+2}{u} \left(\frac{du}{2x+1}\right)$

$= \int \frac{du}{u} \left(\frac{4x+2}{2x+1}\right)$

$= \int \frac{du}{u} \left(\frac{2(2x+1)}{2x+1}\right)$

$= 2 \int \frac{du}{u}$

$= 2\ln |u| + C$

$= 2 \ln | x^2 + x + 5 | + C$

INTEGRAL CALCULUS

3 $\int \frac{3x}{1+9x^2} dx$

$u = 1 + 9x^2 \quad du = 18x\, dx$

$dx = \frac{du}{18x}$

$= \int \frac{3x}{u} \left(\frac{du}{18x}\right)$

$= \frac{3}{18} \int \frac{du}{u}$

$= \frac{1}{6} \ln |u| + C$

$= \frac{1}{6} \ln |1 + 9x^2| + C$

Practice sets

Generalized power rule-u substitution

1. $\int (4x-3)^9\,dx$

2. $\int t\sqrt{7t^2+12}\,dt$

3. $\int \frac{6}{(1-2x)^3}\,dx$

4. $\int \frac{x^3}{(5x^4+2)^3}\,dx$

5 $\int 4\sqrt{5+9t}+12(5+9t)^7\,dt$.

logarithm

6. $\int \frac{t+1}{t}\,dt$

7. $\int \frac{3x^4+12x^3+6}{x^5+5x^4+10x+12}\,dx$

8 $\int \frac{x\,dx}{a^2+x^2}$.

9 $\int \frac{(1-2x)^2}{x}$dx

10 $\int \frac{x^2-x}{x+1}\,dx.$