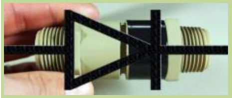
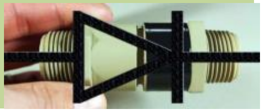
LIMITS AT INFINITY

PRESENTED BY ENGR. JOHN R. REJANO,ECE



OBJECTIVES



- Discus infinity
- Discus limits approaching infinity

INFINITY rules

addition

$$\infty + a = \infty \quad \text{where } a \neq -\infty$$

$$\infty + \infty = \infty$$

subtraction

$$-\infty - a = -\infty \quad \text{where } a \neq -\infty$$

$$-\infty - \infty = -\infty$$

$\infty - \infty$ = Indeterminate form

multiplication

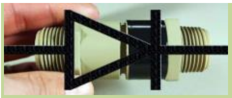
$$(a)(\infty) = \infty \quad \text{if } a > 0 \qquad (a)(\infty) = -\infty \quad \text{if } a < 0$$

$$(\infty)(\infty) = \infty \qquad (-\infty)(-\infty) = \infty \qquad (-\infty)(\infty) = -\infty$$

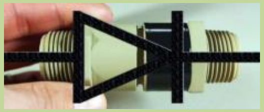
division

$$\frac{\infty}{a} = \infty \quad \text{if } a > 0, a \neq \infty \qquad \frac{\infty}{a} = -\infty \quad \text{if } a < 0, a \neq -\infty$$

$$\frac{-\infty}{a} = -\infty \quad \text{if } a > 0, a \neq \infty \qquad \frac{-\infty}{a} = \infty \quad \text{if } a < 0, a \neq -\infty$$



LIMITS



<u>division</u>

Division of a number by infinity is somewhat intuitive, but there are a couple of subtleties that you need to be aware of. When we talk about division by infinity we are really talking about a limiting process in which the denominator is going towards infinity. So, a number that isn't too large divided an increasingly large number is an increasingly small number. In other words, in the limit we have,

$$\frac{a}{\infty} = 0 \qquad\qquad \frac{a}{-\infty} = 0$$

LIMITS

division

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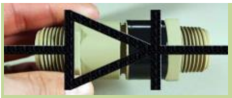
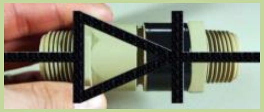


LIMITS



Infinity to the Power Zero

Infinity to the power zero is an indeterminate form:

$$\infty^0 = \text{Indeterminate form}$$

Zero to the Power of a Number

If the power of zero is greater than zero, then the result is zero:

$$0^k = 0, \text{ where k is greater than zero}$$

If the power of zero is greater than infinity, then the result is infinity:

$$0^k = \infty, \text{ where k is less than zero}$$

If the number is greater than one, then the result is infinity:

$$k^\infty = \infty, \text{ where k is greater than 1}$$

If the number is greater than zero but less than one, then the result is zero:

$$k^\infty = 0, 0 < k < 1$$

LIMITS

Zero to the power infinity is equal to zero:

$$0^{\infty} = 0$$

Infinity to the power infinity is equal to infinity:

$$\infty^{\infty} = \infty$$

One to the power infinity results in an indeterminate form:

1^{∞} = Indeterminate form

LIMITS

LIMITS AT INFINITY AND HORIZONTAL ASYMPTOTES

If the values of a variable x increase without bound, then we write $x \to +\infty$, and if the values of x decrease without bound, then we write $x \to -\infty$. The behavior of a function $f(x)$ as x increases without bound or decreases without bound is sometimes called the *end behavior* of the function. For example,

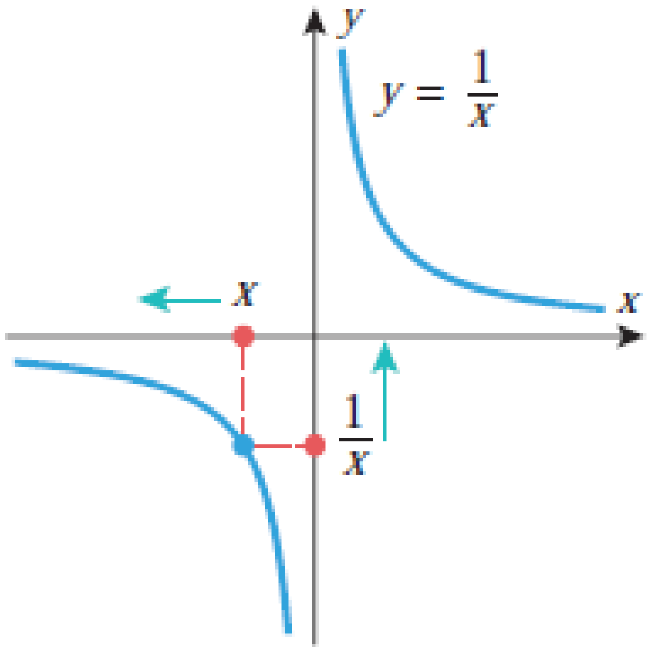
$$\lim_{x \to -\infty} \frac{1}{x} = 0 \quad \text{and} \quad \lim_{x \to +\infty} \frac{1}{x} = 0 \tag{1–2}$$

are illustrated numerically in Table 1.3.1 and geometrically in Figure 1.3.1.

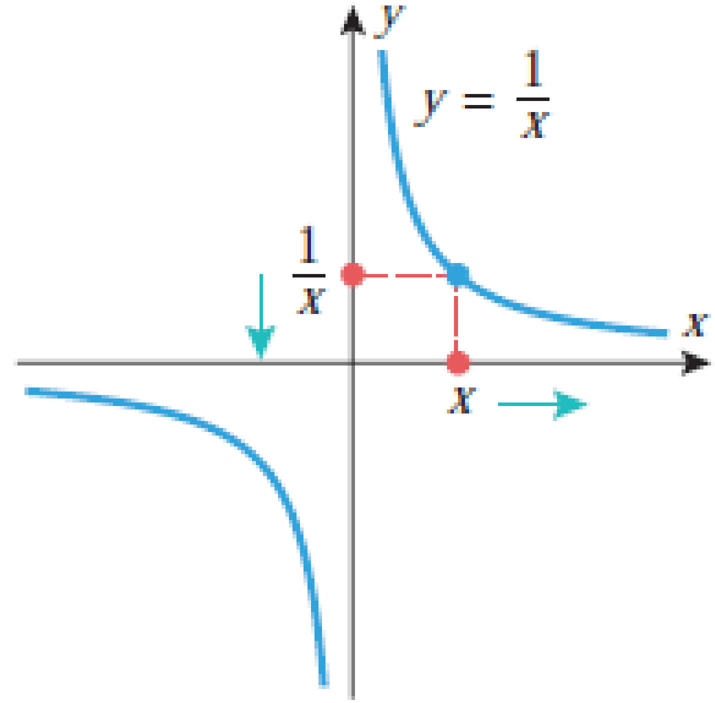
Table 1.3.1

	VALUES						CONCLUSION
x $1/x$	−1 −1	−10 −0.1	−100 −0.01	−1000 −0.001	−10,000 −0.0001	· · · · · ·	As $x \to -\infty$ the value of $1/x$ increases toward zero.
x $1/x$	1 1	10 0.1	100 0.01	1000 0.001	10,000 0.0001	· · · · · ·	As $x \to +\infty$ the value of $1/x$ decreases toward zero.

LIMITS



$$\lim_{x \to -\infty} \frac{1}{x} = 0$$



$$\lim_{x \to +\infty} \frac{1}{x} = 0$$

LIMITS

1.3.1 LIMITS AT INFINITY (AN INFORMAL VIEW) If the values of $f(x)$ eventually get as close as we like to a number L as x increases without bound, then we write

$$\lim_{x \to +\infty} f(x) = L \quad \text{or} \quad f(x) \to L \text{ as } x \to +\infty \tag{3}$$

Similarly, if the values of $f(x)$ eventually get as close as we like to a number L as x decreases without bound, then we write

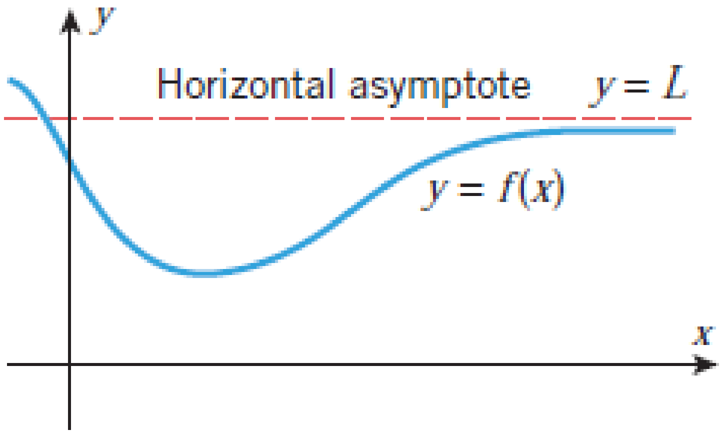
$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad f(x) \to L \text{ as } x \to -\infty \tag{4}$$

Figure 1.3.2 illustrates the end behavior of a function f when

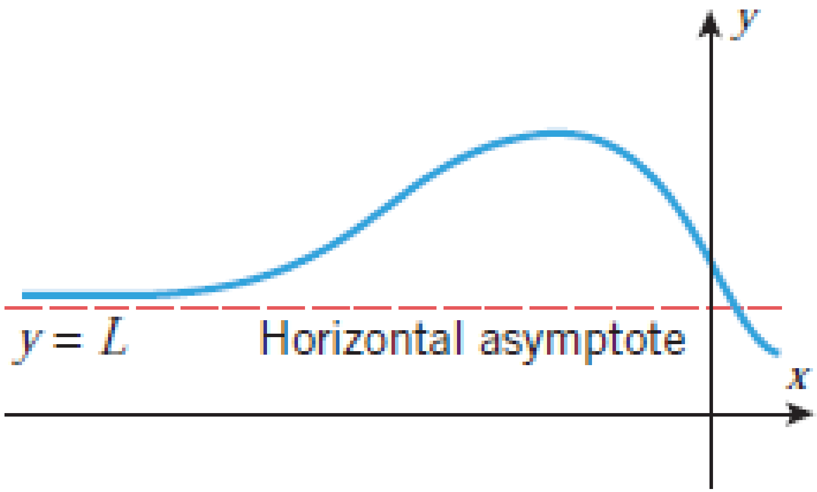
$$\lim_{x \to +\infty} f(x) = L \quad \text{or} \quad \lim_{x \to -\infty} f(x) = L$$

In the first case the graph of f eventually comes as close as we like to the line $y = L$ as x increases without bound, and in the second case it eventually comes as close as we like to the line $y = L$ as x decreases without bound. If either limit holds, we call the line $y = L$ a ***horizontal asymptote*** for the graph of f.

LIMITS



$$\lim_{x \to +\infty} f(x) = L$$

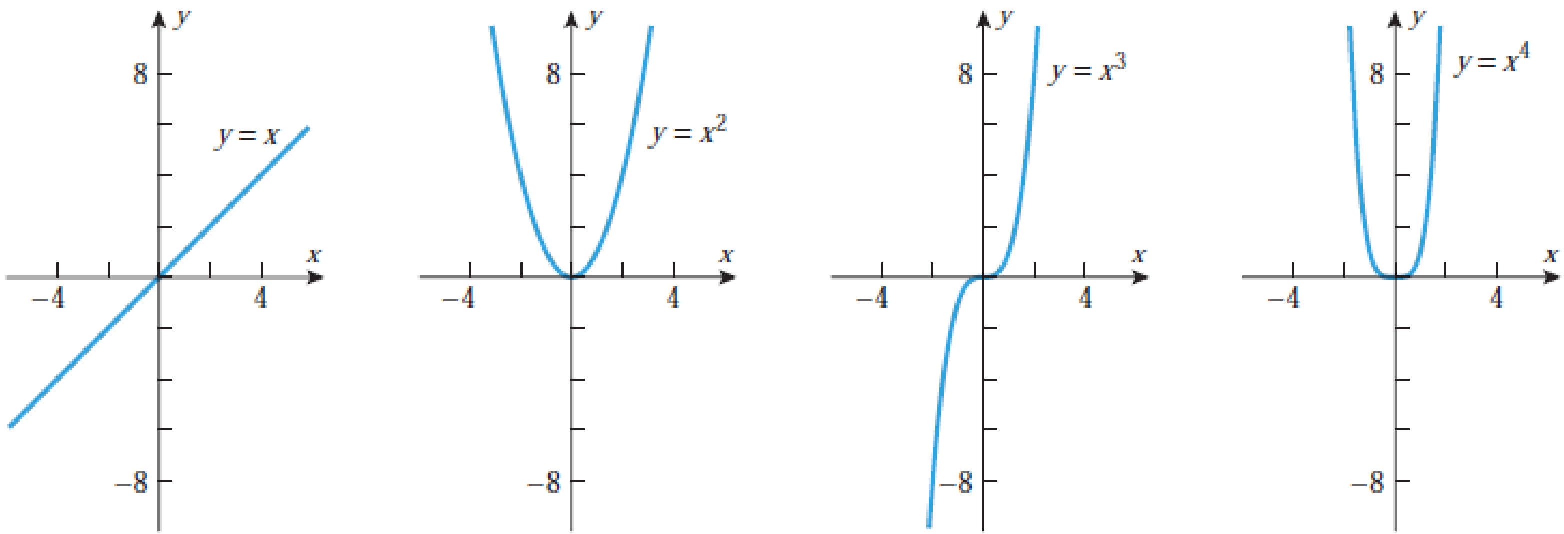


$$\lim_{x \to -\infty} f(x) = L$$

LIMITS OF x^n AS $x \to \pm\infty$

Figure 1.3.5 illustrates the end behavior of the polynomials x^n for $n = 1, 2, 3$, and 4. These are special cases of the following general results:

$$\lim_{x \to +\infty} x^n = +\infty, \quad n = 1, 2, 3, \ldots \qquad \lim_{x \to -\infty} x^n = \begin{cases} -\infty, & n = 1, 3, 5, \ldots \\ +\infty, & n = 2, 4, 6, \ldots \end{cases} \qquad (15\text{–}16)$$



$$\lim_{x \to +\infty} x = +\infty \qquad \lim_{x \to -\infty} x = -\infty$$

$$\lim_{x \to +\infty} x^2 = +\infty \qquad \lim_{x \to -\infty} x^2 = +\infty$$

$$\lim_{x \to +\infty} x^3 = +\infty \qquad \lim_{x \to -\infty} x^3 = -\infty$$

$$\lim_{x \to +\infty} x^4 = +\infty \qquad \lim_{x \to -\infty} x^4 = +\infty$$

▲ **Figure 1.3.5**

LIMITS

Example

$$\lim_{x \to +\infty} 2x^5 = +\infty, \qquad \lim_{x \to -\infty} 2x^5 = -\infty$$

$$\lim_{x \to +\infty} -7x^6 = -\infty, \qquad \lim_{x \to -\infty} -7x^6 = -\infty$$

LIMITS OF POLYNOMIALS AS $x \to \pm\infty$

There is a useful principle about polynomials which, expressed informally, states:

The end behavior of a polynomial matches the end behavior of its highest degree term.

More precisely, if $c_n \neq 0$, then

$$\lim_{x \to -\infty} (c_0 + c_1 x + \cdots + c_n x^n) = \lim_{x \to -\infty} c_n x^n$$

$$\lim_{x \to +\infty} (c_0 + c_1 x + \cdots + c_n x^n) = \lim_{x \to +\infty} c_n x^n$$

LIMITS

Example

$$\lim_{x \to -\infty} (7x^5 - 4x^3 + 2x - 9) = \lim_{x \to -\infty} 7x^5 = -\infty$$

$$\lim_{x \to -\infty} (-4x^8 + 17x^3 - 5x + 1) = \lim_{x \to -\infty} -4x^8 = -\infty$$

LIMITS

LIMITS OF RATIONAL FUNCTIONS AS $x \to \pm\infty$

One technique for determining the end behavior of a rational function is to divide each term in the numerator and denominator by the highest power of x that occurs in the denominator, after which the limiting behavior can be determined using results we have already established. Here are some examples.

Example

Find $\displaystyle\lim_{x \to +\infty} \frac{3x+5}{6x-8}$.

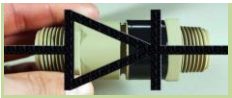
Limits

$$(c) \qquad \lim_{x \to \infty} \frac{2x^4 - 3x^2 + 1}{6x^4 + x^3 - 3x}$$

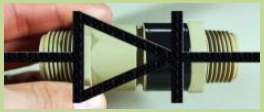
LIMITS

Other examples

$$(a)\ \lim_{x \to \infty} \frac{2x+3}{4x-5} \qquad (b)\ \lim_{x \to \infty} \frac{2x^2+1}{6+x-3x^2} \qquad (c)\ \lim_{x \to \infty} \frac{x}{x^2+5}$$



REFERENCE



1. CALCULUS by H. Anton, et al 10th edition
2. Schaum’s outline series CALCULUS 6th edition by Ayres/ Mendelson